Real-time estimation of interaction delays

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(Received 7 June 2009; published 14 September 2009)

We introduce a real-time method for the identification of time-varying interaction delays in coupled systems with and without unknown structural parameters and analyze the convergence of the delay identification process. The identification method can be applied to monitor the change in interaction delays, as well as to decode the encoding of interaction delays. Several examples are presented to illustrate the reliability and robustness of the suggested strategy.

DOI: 10.1103/PhysRevE.80.036203 PACS number(s): 05.45.Xt

Interaction delays are a fundamental feature that has been observed in various natural systems and may influence their cooperative dynamics behaviors dramatically. The list of typical examples includes living coupled oscillators [1], neurons [2–4], laser systems [5], and gene regulation networks [6], to name just a few. Interaction delays usually vary with time due to some restrictions of signal "transmission" and may even jump from one scale to another. Interaction delays in analog (electronic) neural networks [7], for instance, are time-varying because of the finite switching speed of amplifiers. To describe and understand dynamical behavior and information processes of these coupled systems, we have to take time-varying interaction delays into account and identify them real timely. In this paper, we suggest a real-time method to estimate time-varying interaction delays of coupled systems with and without unknown structural parameters. Several delay identification methods [8–14] have been developed, but they are non-real-time and are applicable only under the assumption that interaction delays are constant (i.e., invariant with the time).

To demonstrate the technique to be introduced in this paper, we analyze coupled systems with a quite general form given by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_1(\boldsymbol{x}) + \boldsymbol{h}_1(\boldsymbol{y}_{\tau_2^*} - \boldsymbol{x}),$$

$$\dot{\boldsymbol{y}} = \boldsymbol{f}_2(\boldsymbol{y}) + \boldsymbol{h}_2(\boldsymbol{x}_{\tau_1^*} - \boldsymbol{y}), \tag{1}$$

where $\boldsymbol{x}=[x_1,x_2,\ldots,x_n]^\top \in \mathbb{R}^n$ and $\boldsymbol{y}=[y_1,y_2,\ldots,y_n]^\top \in \mathbb{R}^n$ are state vectors; $\boldsymbol{f}_1=[f_{11},f_{12},\ldots,f_{1n}]^\top$ and $\boldsymbol{f}_2=[f_{21},f_{22},\ldots,f_{2n}]^\top$ describe the dynamics; $\boldsymbol{h}_1=[h_{11},h_{12},\ldots,h_{1n}]^\top:\mathbb{R}^n\to\mathbb{R}^n$ and $\boldsymbol{h}_2=[h_{21},h_{22},\ldots,h_{2n}]^\top:\mathbb{R}^n\to\mathbb{R}^n$ are coupling functions. We contain time-varying delay $\tau_i^*(t)$ for each coupling with $x_{1,\tau^*}(t) := x_1(t-\tau^*)$. We assume that $\mathbf{s}=\mathbf{s}(\boldsymbol{x},\boldsymbol{y})$ is the ℓ-dimensional experimental measurable output of the coupled system. The main goal considered here is to estimate delays $\tau_1^*(t)$ and $\tau_2^*(t)$ from the measurable output. In the following, we always assume that the information about interaction delays is (implicitly) contained in the output time series [15].

To estimate the values of the delays $\tau_i^*(t)$, we consider the following equations as a "computational model"

$$\dot{\boldsymbol{u}} = \boldsymbol{f}_1(\boldsymbol{u}) + \boldsymbol{h}_1(\mathbf{v}_{\tau_2} - \boldsymbol{u}) + \mathbf{w}_1(\mathbf{s},\boldsymbol{u},\mathbf{v}),$$

$$\dot{\mathbf{v}} = \boldsymbol{f}_2(\mathbf{v}) + \boldsymbol{h}_2(\boldsymbol{u}_{\tau_1} - \mathbf{v}) + \mathbf{w}_2(\mathbf{s},\boldsymbol{u},\mathbf{v}),$$

$$\dot{\tau}_1 = g_1(\boldsymbol{u},\mathbf{v},\boldsymbol{u}_{\tau_1},\mathbf{v}_{\tau_2}),$$

$$\dot{\tau}_2 = g_2(\boldsymbol{u},\mathbf{v},\boldsymbol{u}_{\tau_1},\mathbf{v}_{\tau_2}), \tag{2}$$

where the stream of the measurable output $\mathbf{s}(\boldsymbol{x},\boldsymbol{y})$ from system (1) is used to dynamically change the model parameters and the model state. Here $\tau_i(t)$ is an estimation of true delay $\tau_i^*(t)$; and control signals $\mathbf{w}_i$ and functions g_i will be specified below such that the model is applicable to estimate true delay $\tau_i^*(t)$ with acceptable accuracy.

Let $\mathbf{e}_1(t) := \boldsymbol{u}(t) - \boldsymbol{x}(t)$, $\mathbf{e}_2(t) := \mathbf{v}(t) - \boldsymbol{y}(t)$, $\lambda_i(t) := \tau_i(t) - \tau_i^*(t)$, and $\mathbf{e}_i=[e_{i1},e_{i2},\ldots,e_{in}]^\top$. Then the error system reads

$$\dot{e}_{1k} = h_{1k}(\mathbf{v}_{\tau_2} - \boldsymbol{u}) - h_{1i}(\boldsymbol{y}_{\tau_2^*} - \boldsymbol{x}) + f_{1k}(\boldsymbol{u}) - f_{1k}(\boldsymbol{x}) + w_{1k}, \quad \forall\ k=1,2,\ldots,n,$$

$$\dot{e}_{2k} = h_{2k}(\boldsymbol{u}_{\tau_1} - \mathbf{v}) - h_{2k}(\boldsymbol{x}_{\tau_1^*} - \boldsymbol{y}) + f_{2k}(\mathbf{v}) - f_{2k}(\boldsymbol{y}) + w_{21}, \quad \forall\ k=1,2,\ldots,n,$$

$$\dot{\lambda}_i = g_i - \dot{\tau}_i^*, \quad \forall\ i=1,2. \tag{3}$$

After all delays are identified [i.e., $\tau_i(t)=\tau_i^*(t)$, $\forall i$], error system (3) actually reads

$$\dot{e}_{1k} = h_{1k}(\mathbf{v}_{\tau_2^*} - \boldsymbol{u}) - h_{1i}(\boldsymbol{y}_{\tau_2^*} - \boldsymbol{x}) + f_{1k}(\boldsymbol{u}) - f_{1k}(\boldsymbol{x}) + w_{1k}, \quad \forall\ k=1,2,\ldots,n,$$

$$\dot{e}_{2k} = h_{2k}(\boldsymbol{u}_{\tau_1^*} - \mathbf{v}) - h_{2k}(\boldsymbol{x}_{\tau_1^*} - \boldsymbol{y}) + f_{2k}(\mathbf{v}) - f_{2k}(\boldsymbol{y}) + w_{21}, \quad \forall\ k=1,2,\ldots,n. \tag{4}$$

Therefore the necessary condition for ensuring a successful delay identification is that one can find some proper control signals $\mathbf{w}_i$ such that the error system (4) is asymptotically stable. The issue is well studied in the literature and can be attacked by using tools such as conditional Lyapunov exponents approach and Lyapunov's direct method. For instance, by following the work of Ref. [16], we can easily show that if $\mathbf{w}_1=-k_1(\boldsymbol{u}-\boldsymbol{x})$ and $\mathbf{w}_2=-k_2(\mathbf{v}-\boldsymbol{y})$ (with sufficient high gains k_1 and k_2) are used, then the following Lyapunov function

$$V_o=\frac{1}{2}\left[\sum_k e_{1k}^2+\sum_k e_{2k}^2+L_{21}\int_{t-\tau_2^*}^{t}\sum_k e_{1k}(\eta)^2 d\eta + L_{22}\int_{t-\tau_1^*}^{t}\sum_k e_{2k}(\eta)^2 d\eta\right] \tag{5}$$

decreases monotonously along the trajectories of error system (4). This implies $\dot V_o|_{\mathrm{Eq.(4)}}\le 0$ and $\mathbf{e}_i\to\mathbf{0}$, $\forall\ i$.

However, even when one can find some proper control signals $\mathbf{w}_i$ such that the error system (4) is asymptotically stable, it is easy to see from Eq. (3) that the difference between $\tau_i(t)$ and their true values $\tau_i^*(t)$ in general will destroy the synchronization between system (1) and its model [Eq. (2)]. To preserve the synchronization, one has to choose the functions g_i carefully in order to compensate for (or eliminate) the error caused by the difference between $\tau_i(t)$ and their true values $\tau_i^*(t)$.

For this purpose, for the error system (3) we choose a Lyapunov function

$$V(\mathbf{e}_1,\mathbf{e}_2,\lambda_1,\lambda_2)=V_o+\lambda_1^2/(2\delta_1)+\lambda_2^2/(2\delta_2), \tag{6}$$

where δ_i are positive and V_o is defined by Eq. (5).

Differentiating V along the trajectories of system (3), one gets

$$\dot V|_{\mathrm{Eq.(3)}}=\dot V_o|_{\mathrm{Eq.(4)}}+\dot\lambda_1\lambda_1/\delta_1+\dot\lambda_2\lambda_2/\delta_2+\sum_k e_{1k}[h_{1k}(\mathbf{v}_{\tau_2}-\boldsymbol{u}) - h_{1k}(\mathbf{v}_{\tau_2^*}-\boldsymbol{u})]+\sum_k e_{2k}[h_{2k}(\boldsymbol{u}_{\tau_1}-\mathbf{v})-h_{2k}(u_{\tau_1^*}-\mathbf{v})], \tag{7}$$

where the last two terms of the right-hand side come from the difference between systems (3) and (4), which will be eliminated by designing proper functions g_i (due to $\dot\lambda_i=g_i-\dot\tau_i^*\ \forall\ i$).

By considering $h_{1k}(\mathbf{v}_{\tau_2^*}-\boldsymbol{u})$ as a function of τ_2^* and expanding $h_{1k}(\mathbf{v}_{\tau_2^*}-\boldsymbol{u})$ at $\tau_2^*=\tau_2$, one obtains the following first-order approximation

$$h_{1k}(\mathbf{v}_{\tau_2^*}-\boldsymbol{u})\approx h_{1k}(\mathbf{v}_{\tau_2}-\boldsymbol{u})-\frac{\partial h_{1k}(\mathbf{v}_{\tau_2}-u_1)}{\partial\tau_2}\lambda_2. \tag{8}$$

Similarly one achieves

$$h_{2k}(\boldsymbol{u}_{\tau_1^*}-\mathbf{v})\approx h_{2k}(\boldsymbol{u}_{\tau_1}-\mathbf{v})-\frac{\partial h_{2k}(\boldsymbol{u}_{\tau_1}-\mathbf{v})}{\partial\tau_1}\lambda_1. \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (7) yields

$$\dot V|_{\mathrm{Eq.(3)}}=\dot V_o|_{\mathrm{Eq.(4)}}+\dot\lambda_1\lambda_1/\delta_1+\dot\lambda_2\lambda_2/\delta_2 +\lambda_2\sum_k\frac{\partial h_{1k}(\mathbf{v}_{\tau_2}-\boldsymbol{u})}{\partial\tau_2}e_{1k}+\lambda_1\sum_k\frac{\partial h_{2k}(\boldsymbol{u}_{\tau_1}-\mathbf{v})}{\partial\tau_1}e_{2k}. \tag{10}$$

To eliminate the last two terms of the right-hand side of the above equation, one can design

$$g_1=-\delta_1\sum_k\frac{\partial h_{2k}(\boldsymbol{u}_{\tau_1}-\mathbf{v})}{\partial\tau_1}e_{2k},$$

$$g_2=-\delta_2\sum_k\frac{\partial h_{1k}(\mathbf{v}_{\tau_2}-\boldsymbol{u})}{\partial\tau_2}e_{1k}, \tag{11}$$

which indicates

$$\dot V|_{\mathrm{Eq.(3)}}=\dot V_o|_{\mathrm{Eq.(4)}}-\dot\tau_1^*\lambda_1/\delta_1-\dot\tau_2^*\lambda_2/\delta_2. \tag{12}$$

If all delays τ_i^* are constant ($\dot\tau_i^*(t)=0\,\forall\, i$), then $\dot V|_{\mathrm{Eq.(3)}}=\dot V_o|_{\mathrm{Eq.(4)}}$ which implies from $\dot V_o|_{\mathrm{Eq.(4)}}\le 0$ (shown above) that all state vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ as well as delays τ_i^* can be estimated correctly. When sufficiently large δ_i are used, $\dot V_o|_{\mathrm{Eq.(4)}}$ dominates the right-hand side of Eq. (12) and hence all time-varying delays τ_i^* can also be estimated with acceptable accuracy.

Let us now move to show the reliability of the proposed technique by making reference to a series of examples. As a first example, we analyze the case of coupled identical Rössler systems:

$$\begin{aligned}
\dot x_1&=-x_2-x_3,\\
\dot x_2&=x_1+ax_2+0.1(y_{2,\tau_2^*}-x_2),\\
\dot x_3&=b+(x_1-c)x_3,\\
\dot y_1&=-y_2-y_3,\\
\dot y_2&=y_1+ay_2+0.1(x_{2,\tau_1^*}-y_2),\\
\dot y_3&=b+(y_1-c)y_3,
\end{aligned} \tag{13}$$

where parameters are set to be $a=b=0.2$ and $c=5.7$. As a model, we consider Eq. (2) with $n=3$; $w_{1i}=-10(u_i-x_i)$ and $w_{2i}=-10(v_i-y_i)$ for $i=1,2$; $w_{13}=w_{23}=0$; $g_1=10\dot u_{2,\tau_1}\ (v_2-y_2)$ and $g_2=10\dot v_{2,\tau_2}\ (u_2-x_2)$.

Figure 1 shows that periodically time-varying interaction delays can be identified with high accuracy [here we display only the case that $\tau_1^*(t)$ is sinusoidal and $\tau_2^*(t)\equiv 2$]. Now we consider a scenario in which each Rössler system corresponds to a neuron and the change in interaction delays corresponds to a "neural encoding" processing. To understand this information processing, we have to decode all encodings. More precisely, we have to monitor the change in interaction delays real timely. As a typical example Fig. 2 displays a "binary" delay-encoding processing and shows that

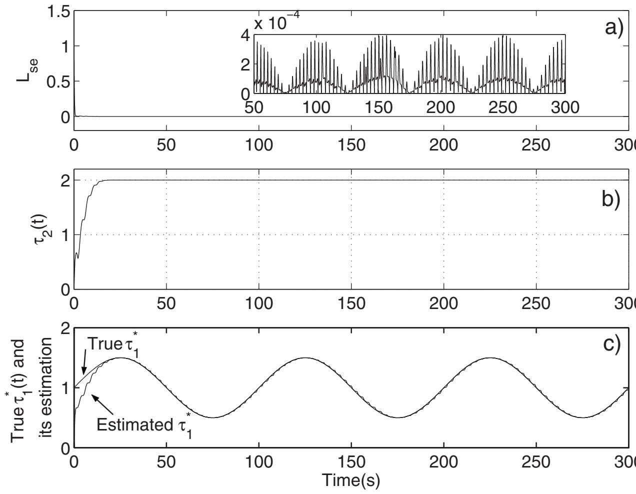

FIG. 1. Identification of sinusoidally time-varying delays. (a) Root-mean-square error of state estimation $L_{se}(t)$ (see inset for its partial enlarged drawing) vs time. (b) Estimation value $\tau_2(t)$. (c) True $\tau_1^*(t)$ and its estimation $\tau_1(t)$ vs time.

all encoding information can be decoded with high accuracy (here we display the case that only $\tau_1^*(t)$ is used for encoding).

Noise exists in many natural coupled systems, which can arise from either intrinsic sources or extrinsic sources which are attributable to a noisy environment. It is, therefore, of great value to analyze the influence of noise on the robustness of delay identification. Figure 3 summarizes our numerical results. It is easy to see from Inserts of Fig. 3(b) that estimated delays fluctuate around their true values. However, by using proper filter techniques (here we applied average filters), we can still estimate delays with high accuracy [cf. insets of Fig. 3(c)].

It should be noticed that the model (used for Figs. 1–3) applies only states x_1, x_2, y_1, and y_2. This implies that it is still possible to estimate unknown interaction delays by ex-

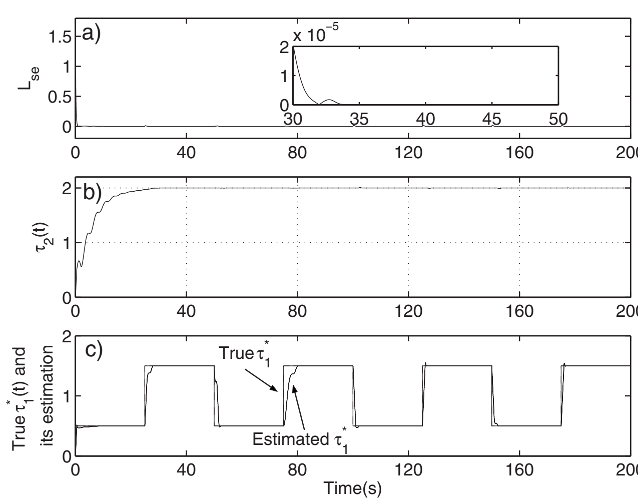

FIG. 2. Decoding binary delay encoding. (a) Root-mean-square error of state estimation $L_{se}(t)$ (see inset for its partial enlarged drawing) vs time. (b) Estimation value $\tau_2(t)$. (c) True $\tau_1^*(t)$ and its estimation $\tau_1(t)$ vs time.

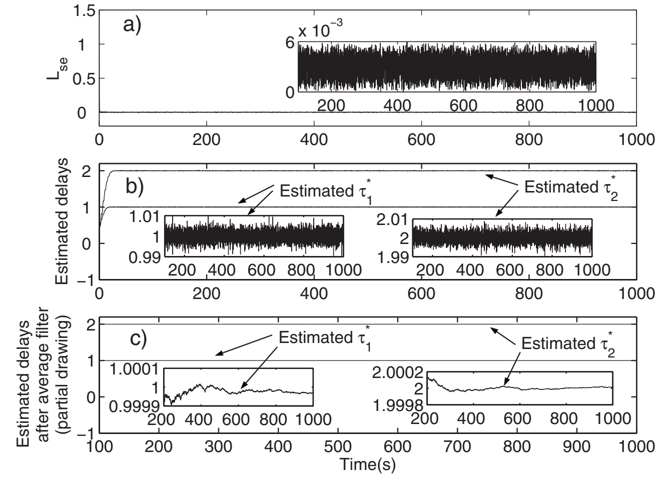

FIG. 3. Delay identification in the presence of measurement noise uniformly distributed in the range $[-0.01,+0.01]$. (a) Root-mean-square error of state estimation $L_{se}(t)$ (see inset for its partial enlarged drawing) vs time. (b) Estimation values $\tau_1(t)$ (see left inset for its partial enlarged drawing) and $\tau_2(t)$ (see right inset for its partial enlarged drawing) vs time. (c) Estimation values $\tau_1(t)$ and $\tau_2(t)$ after average filter and their partial enlarged drawings (see insets).

ploiting information obtained from only partial state variables of coupled systems.

The above analysis focuses on delay estimation under the reasonable conditions that the coupling functions and the local dynamics of each element are known. If the coupling functions and the local dynamics of each element are unknown, one can follow the work of Ref. [17] and estimate the coupling functions and the local dynamics of each element with arbitrarily high accuracy by driving the coupled system to distinct stationary states. One can also attack this problem by using the strategy (as shown in Figs. 1–3) in combination with the adaptive parameter estimation strategy [18]. This will be demonstrated as follows.

It is reasonable to assume that

$$f_{ij}(z) \approx \sum_k p_{1ijk}\delta_{ijk}(z),$$

$$h_{ij}(z) \approx \sum_k p_{2ijk}\eta_{ijk}(z), \tag{14}$$

where $\delta_{ijk}(\boldsymbol{x})$ and $\eta_{ijk}(\boldsymbol{x})$ are taken from kernel (or orthogonal basic) functions set (e.g., polynomial functions set). One can achieve improved accuracy when higher order kernel functions are contained in f_{ij} and h_{ij}.

To identify interactions of system (1) under condition (14), we also have to estimate those unknown structural parameters p_{1ijk} and p_{2ijk}. For this purpose, the functions f_{ij} and h_{ij} in model (2) now read

$$f_{ij}(z) \approx \sum_k q_{1ijk}\delta_{ijk}(z),$$

$$h_{ij}(z) \approx \sum_k q_{2ijk}\eta_{ijk}(z), \tag{15}$$

where parameters q_{1ijk} and q_{2ijk} are used to estimate their true values p_{1ijk} and p_{2ijk} and are updated by $\dot{q}_{1ijk}=\phi_{1ijk}(\mathbf{s},\boldsymbol{u},\mathbf{v})$ and $\dot{q}_{2ijk}=\phi_{2ijk}(\mathbf{s},\boldsymbol{u},\mathbf{v})$. The functions ϕ_{1ijk} and ϕ_{2ijk} can be determined by the well-developed adaptive observer technique [18] and can simply be designed as $\phi_{1ijk}=-\delta_{ijk}(\boldsymbol{u})(u_j-x_j)$, $\phi_{2ijk}=-\eta_{ijk}(\boldsymbol{u})(u_j-x_j)$, for $i=1$; $\phi_{1ijk}=-\delta_{ijk}(\mathbf{v})(v_j-y_j)$, $\phi_{2ijk}=-\eta_{ijk}(\mathbf{v})(v_j-y_j)$, for $i=2$. Substituting Eq. (15) into Eq. (11), one can easily design proper functions g_1 and g_2.

To illustrate this delay identification method of coupled systems with unknown structural parameters, we revisit the coupled Rösser system (13) but assume that the structure of the first Rösser system is unknown and can be approximated by a polynomial series expansion [that is, $\delta_{ijk}(z)$ in Eq. (14) are taken from polynomial functions set]. In our experiment, we assume that the first Rösser system can be described by

$$\dot{x}_1 = -x_1 + p_1x_1 + p_2x_2 + p_3x_3,$$

$$\dot{x}_2 = -x_2 + p_4x_1 + p_5x_2 + p_6x_1x_2,$$

$$\dot{x}_3 = -x_3 + p_7 + p_8x_1 + p_9x_3 + p_{10}x_1x_3 + p_{11}x_1x_2,$$

which contains 11 unknown structural parameters p_i and can model many chaotic systems (including Lorenz and Rössler systems).

Simulation results plotted in Fig. 4 show that unknown delays τ_1^* and τ_2^* can be identified with high accuracy [cf. inserts of Fig. 4(b)]; and 11 unknown structural parameters p_i can also be estimated with high accuracy [cf. insert of Fig. 4(c)].

It should be remarked that the complexity of the adaptive delay estimation algorithm depends on: (i) the nonlinearity (or complexity) of functions f_{ij} and h_{ij}; (ii) the form of kernel functions and the number of the unknown structural parameters contained in f_{ij} or/and h_{ij}; (iii) the number of the delays to be identified; and (iv) the dynamics of the delays to be identified. It becomes very significant to reduce the number of the unknown structural parameters and improve the convergence rate of parameter identification by choosing proper kernel functions. For this reason, radial basis function neural network (RBFN) [19–21] in practice may be used to approximate f_{ij} or/and h_{ij} (i.e., Gaussian functions as kernel functions are used to approximate functions f_{ij} or/and h_{ij}) because it can approximate any regular function with arbitrary accuracy and its training is faster than that of a multilayer perceptron. Even so, there in principle exists a limitation if the number of unknown structural parameters in the RBFN increases gradually; but, in the meanwhile, the information concerning the states of the entire system cannot be (further) added. Detailed analysis deserves a careful investigation and will be reported elsewhere.

Compared with previous non-real-time delay identification methods [8–14], the suggested technique has the following advantages: (i) it can easily be extended to identify multiple time-varying delay parameters; (ii) it does not require signal derivative estimator which is sensitive to noise; (iii) it

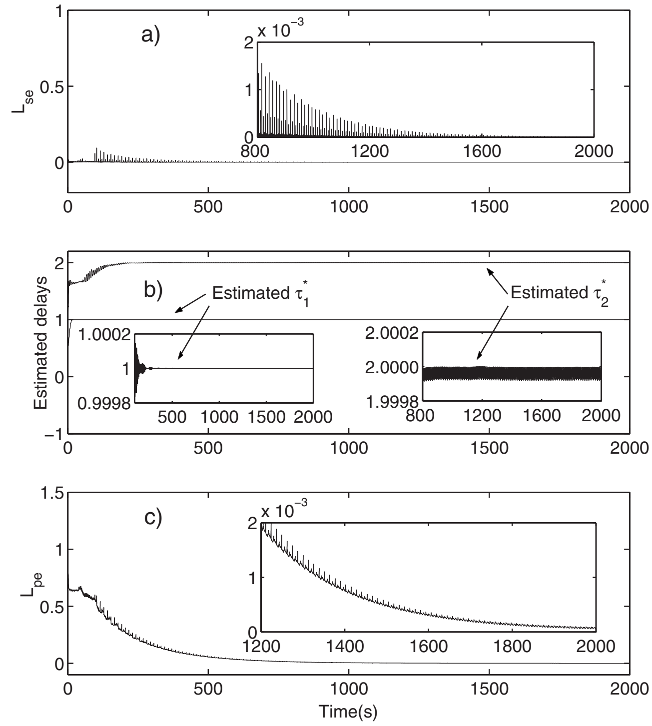

FIG. 4. Delay identification of the coupled system with 11 unknown parameters p_i. (a) Root-mean-square error of state estimation $L_{se}(t)$ (see inset for its partial enlarged drawing) vs time. (b) Estimation values $\tau_1(t)$ (see left inset for its partial enlarged drawing) and $\tau_2(t)$ (see right inset for its partial enlarged drawing) vs time. (c) Parameter estimation error $L_{pe}(t)=\Sigma_{i=1}^{11}|q_i-p_i|/11$ (see inset for its partial enlarged drawing) vs time.

can be extended to high dimensional systems.

The suggested delay identification method can be extended to network systems given by

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}_i(\boldsymbol{x}_i) + \sum_{j=1}^{N} \boldsymbol{h}_{ij}(\boldsymbol{y}_{j,\tau_{ij}^*} - \boldsymbol{x}_i), \tag{16}$$

where $i=1,2,\ldots,N$; $\boldsymbol{x}_i=[x_{i1},x_{i2},\ldots,x_{in}]^\top \in \mathbb{R}^n$ is the state vector of the ith node; $\boldsymbol{f}_i=[f_{i1},f_{i2},\ldots,f_{in}]^\top$ describes the dynamics of the ith node; $\boldsymbol{h}_{ij}=[h_{ij1},h_{ij2},\ldots,h_{ijn}]^\top:\mathbb{R}^n\to\mathbb{R}^n$ are coupling functions. We contain time-varying delay $\tau_{ij}^*(t)$ for the coupling from the jth node to the ith node.

As a model, we consider the following equations:

$$\dot{z}_i = \boldsymbol{f}_i(z_i) + \sum_{j=1}^{N} \boldsymbol{h}_{ij}(z_{j,\tau_{ij}} - z_i) + \boldsymbol{u}_i,$$

$$\dot{\tau}_{ij} = g_{ij}(z_i,z_j,\boldsymbol{x}_i), \tag{17}$$

where $i=1,2,\ldots,N$; $z_i=[z_{i1},z_{i2},\ldots,z_{in}]^\top \in \mathbb{R}^n$ is the state vector of the ith node; $\boldsymbol{u}_i$ are control signals to be designed such that $z_i \to \boldsymbol{x}_i$ after all delay parameters τ_{ij}^* have been identified correctly (i.e., $\tau_{ij}=\tau_{ij}^*$); and functions g_{ij} need to be specified below.

Following the similar steps used for achieving Eq. (11), one can easily design proper functions g_{ij} as follows:

$$g_{ij} = -\delta_{ij}\sum_{k}\frac{\partial h_{ijk}(z_{j,\tau_{ij}} - z_i)}{\partial \tau_{ij}}e_{ik}, \qquad (18)$$

where positive constants δ_{ij} are used to improve the convergence rate as well as the estimation accuracy in the case of that time-varying delay identification is required (i.e., τ_{ij}^* are time varying).

In conclusion, we have introduced a real-time method to estimate interaction delays of coupled systems with and without unknown structural parameters. This method can be applied to identify the change in interaction delays and therefore can monitor real-timely any fault (or information processing) leading to some change in interaction delays. Furthermore the proposed delay identification approach is applicable to decode the delay-encoding (although we showed here only binary encoding, our method can be used to more general forms of encoding). In the context of chaos secure communication, the technique allows one to enhance the security if the transmitting message is modulated by delays, which can be recovered by the proposed delay identification method. The suggested interaction delay identification method can be generalized to dynamical networks and it in combination with previous work [22] can be applied to estimate topology of networks with time-varying interacting delays.

Work partly supported by the Chinese National Natural Science Foundation under Grand No. 10602026 and by EU Contract No. 043309 GABA. D.Y. thanks U. Parlitz for helpful discussions.

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